This question paper contains 4 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper: 8372 **HC**

Unique Paper Code : **32357505**

Name of Paper : **Discrete Mathematics**

Name of Course : **Mathematics : DSE for Hons.**

Semester : **V**

Duration : **3 hours**

Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts from each questions.

Section I

1 (a) Define 'covering relation' in an ordered set. Prove that if P and Q are two ordered sets, then (a_2,b_2) covers (a_1,b_1) in $P\times Q$ if and only if either ($a_1=a_2$ and b_2 covers b_1) or (a_2 covers a_1 and $b_1=b_2$). (6)

(b) Let N_0 be the set of whole numbers equipped with the partial order $\le$ defined by $m\le n$ if and only if m divides n. Draw a Hasse diagram and find out maximal and minimal elements, if they exist, for the subset $\{2,3,4,6,10,12,0\}$ of (N_0, $\le$). Does it have the smallest and the greatest elements? Justify your answer. (6)

(c) Define an order isomorphism for ordered sets. Show that every order isomorphism is bijective but the converse is not true. (6)

2 (a) Let $(L,\le)$ be a lattice as an ordered set. Define two binary operations + and $\cdot$ on L by $x+y=x\vee y=\sup\{x,y\}$ and $x\cdot y=x\wedge y=\inf\{x,y\}$. Prove that $(L,+,\cdot)$ is an algebraic lattice. (6.5)

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(b) Let L be a lattice and let $x,y,z\in L$. Prove that

(i) $y\le z\Rightarrow x\wedge y\le x\wedge z$ and $x\vee y\le x\vee z$

(ii) $((x\wedge y)\vee(x\wedge z))\wedge((x\wedge y)\vee(y\wedge z))=x\wedge y$ (6.5)

(c) Let $f:L\to K$ be a lattice homomorphism. Show that

(i) If S is a sublattice of L, then $f(S)$ is a sublattice of K.

(ii) If T is a sublattice of K and $f^{-1}(T)$ is non-empty, then $f^{-1}(T)$ is a sublattice of L. (6.5)

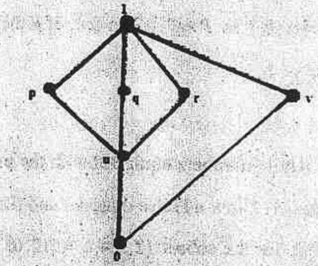
Section II

3 (a) Prove that a lattice L is distributive if and only if $\forall a,b,c\in L$ we have

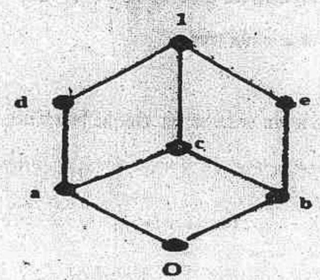
$$(a\vee b=c\vee b \text{ and } a\wedge b=c\wedge b)\Rightarrow a=c.$$

(6)

(b) Use M_3-N_5 Theorem to find if the lattices L_1 and L_2 given below are modular or distributive:



L_1



L_2

(6)

(c) Find the Conjunctive Normal form of

$$(x_1 + x_2 + x_3)(x_1 x_2 + x_1' x_3)'.$$

(6)

4 (a) Define sectionally complemented lattice. Show that every Boolean Algebra is sectionally complemented.

(6.5)

(b) Find all the prime implicants of $xy'z + x'yz' + xyz' + xyz$ and form the corresponding prime implicant table.

(6.5)

(c) Draw the contact diagram and give the symbolic representation of the circuit given by

$$p = (x_1 + x_2 + x_3)(x_1' + x_2)(x_1 x_3 + x_1' x_2)(x_2' + x_3).$$

(6.5)

Section III

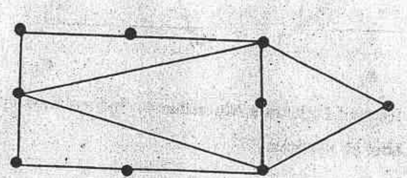
5 (a) (i) Answer the Königsberg bridge problem and explain your answer with graph.

(ii) Draw $K_{3,6}$ and $K_{4,4}$.

(3, 3)

(b) (i) Draw a graph with 5 vertices and as many edges as possible. How many edges does your graph contain. What is the name of this graph and how is it denoted?

(ii) What is bipartite graph? Determine whether the graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite.

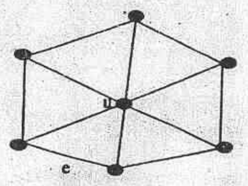


(3, 3)

(c) (i) Draw a graph whose degree sequence is 1,1,1,1,1,1.

(ii) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 4. Justify your answer.

(iii) Draw pictures of the subgraphs G \{e} and G \{u} of the following graph G:

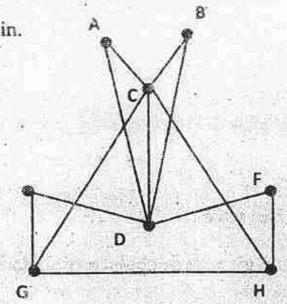


G

(2, 2, 2)

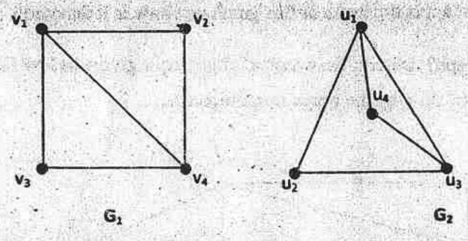
6 (a) (i) Consider the graph G given below. Is it Hamiltonian? If no, explain your answers, if yes find a Hamiltonian cycle.

(ii) Is it Eulerian? Explain.



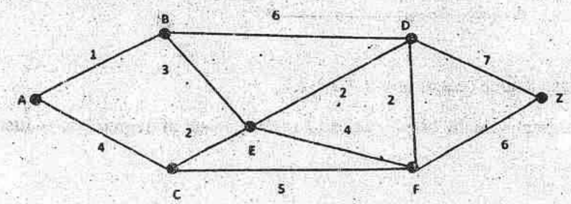
(4, 2.5)

(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$.



(6.5)

(c) Apply the first form of Dijkstra's Algorithm to find a shortest path from A to Z in the graph shown. Label all vertices.



(6.5)

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